

Electrical Technology (EE-101-F)

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Maximum Power Transfer Theorem

⚡ The maximum power transfer theorem states the following:

A load will receive maximum power from a network when its total resistive value is exactly equal to the Thévenin resistance of the network applied to the load. That is,

$$R_L = R_{Th}$$

Maximum Power Transfer Theorem

⚡ For loads connected directly to a dc voltage supply, maximum power will be delivered to the load when the load resistance is equal to the internal resistance of the source; that is, when:

$$R_L = R_{int}$$

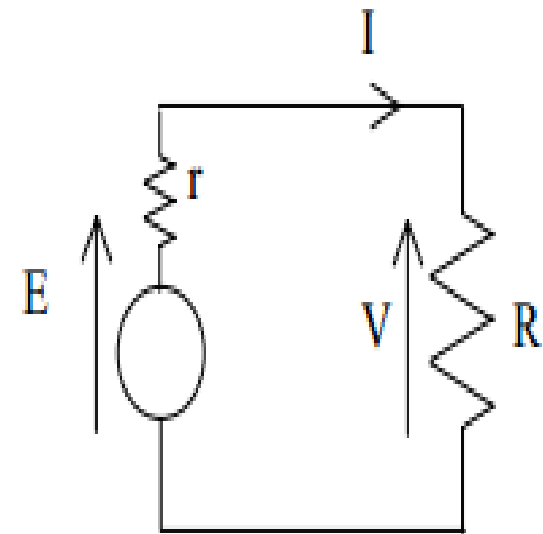
EX Maximum Power Transfer Theorem

Resistive Load supplied from a source with only an internal resistance

Consider a source with an internal emf of E and an internal resistance of r and a load of resistance R .

$$\text{current } I = \frac{E}{R+r}$$

$$\text{Load Power } P = I^2 \cdot R = \left(\frac{E}{R+r} \right)^2 \cdot R$$



The source resistance is dependant purely on the source and is a constant, as is the source emf. Thus only the load resistance R is a variable.

To obtain maximum power transfer to the load, let us differentiate with respect to R.

$$\frac{dP}{dR} = \frac{E^2}{(R+r)^4} \cdot [(R+r)^2 \cdot 1 - R \cdot 2(R+r)] = 0 \text{ for maximum}$$

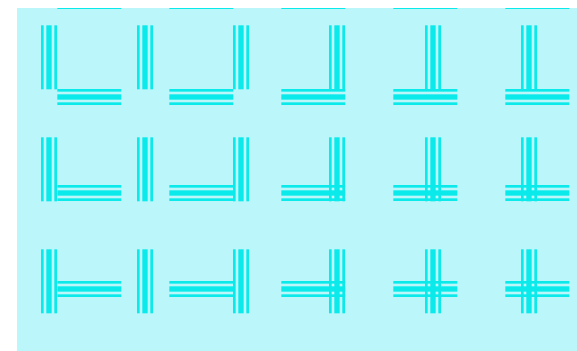
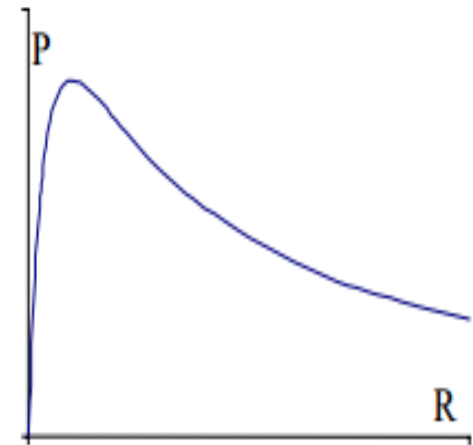
$$\therefore (R+r)^2 - 2R(R+r) = 0$$

$$\text{or } R + r - 2R = 0$$

i.e. $R = r$ for maximum power transfer.

$$\text{value of maximum power} = P_{\max} = \left(\frac{E}{r+r} \right)^2 \cdot r = \frac{E^2}{4r}$$

$$\text{load voltage at maximum power} = \frac{E}{R+r} \cdot R = \frac{E}{r+r} \cdot r = \frac{E}{2}$$



It is to be noted that when maximum power is being transferred, only half the applied voltage is available to the load, and the other half drops across the source. Also, under these conditions, half the power supplied is wasted as dissipation in the source.

– Millman's Theorem

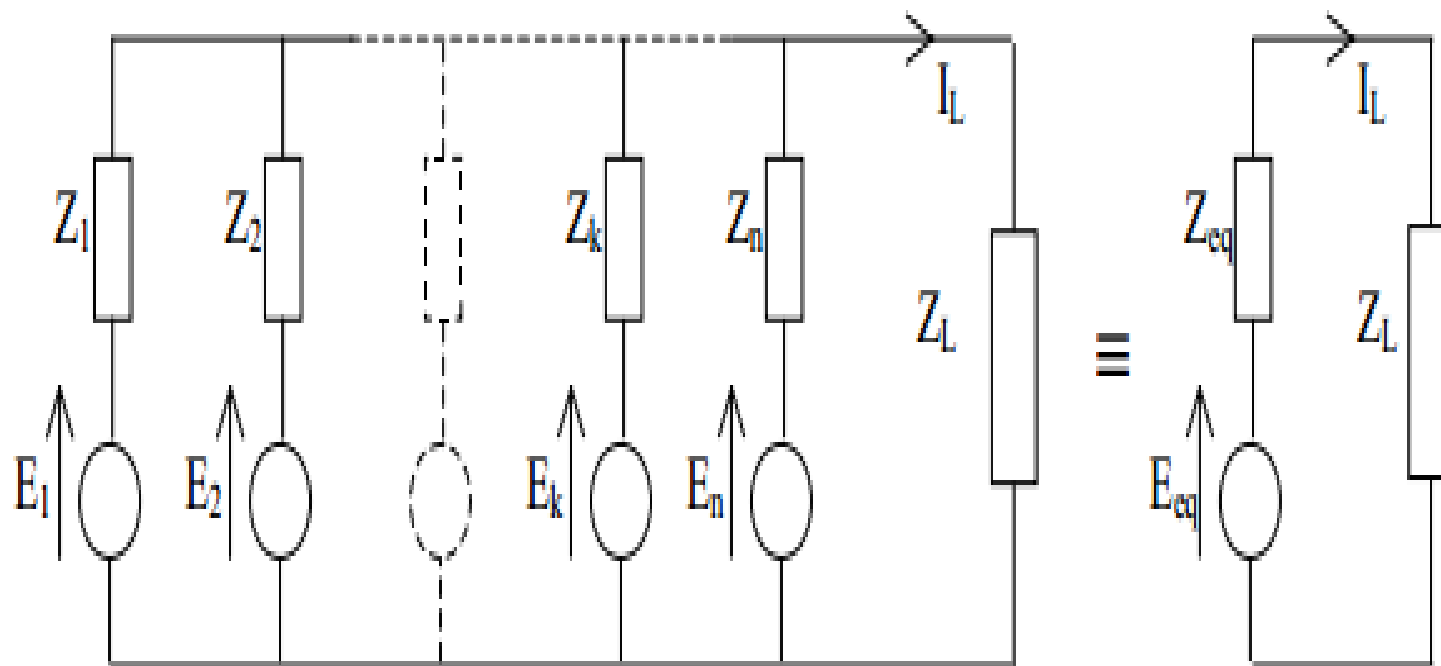
- ⌘ Any number of parallel voltage sources can be reduced to one.
- ⌘ This permits finding the current through or voltage across R_L without having to apply a method such as mesh analysis, nodal analysis, superposition and so on.
 1. Convert all voltage sources to current sources.
 2. Combine parallel current sources.
 3. Convert the resulting current source to a voltage source and the desired single-source network is obtained.

Millman's Theorem

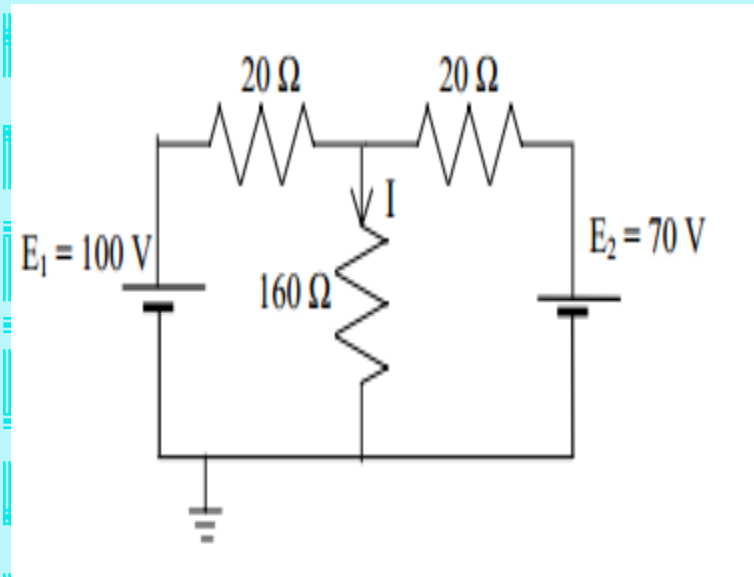
This theorem states that a system of voltage sources operating in parallel may be replaced by a single voltage source in series with an equivalent impedance given as follows (this is effectively the Thevenin's theorem applied to a number of generators in parallel).

$$E_{eq} = \frac{\sum_{k=1}^n E_k Y_k}{\sum_{k=1}^n Y_k}, \quad Y_{eq} = \sum_{k=1}^n Y_k$$

Millman's Theorem



EX Millman's Theorem



Solution

$$E_{\text{eq}} = \frac{\frac{1}{20}100 + \frac{1}{20}70}{\frac{1}{20} + \frac{1}{20}} = 85 \text{ V (same answer was obtained with Thevenin's Th}^{\text{m}})$$

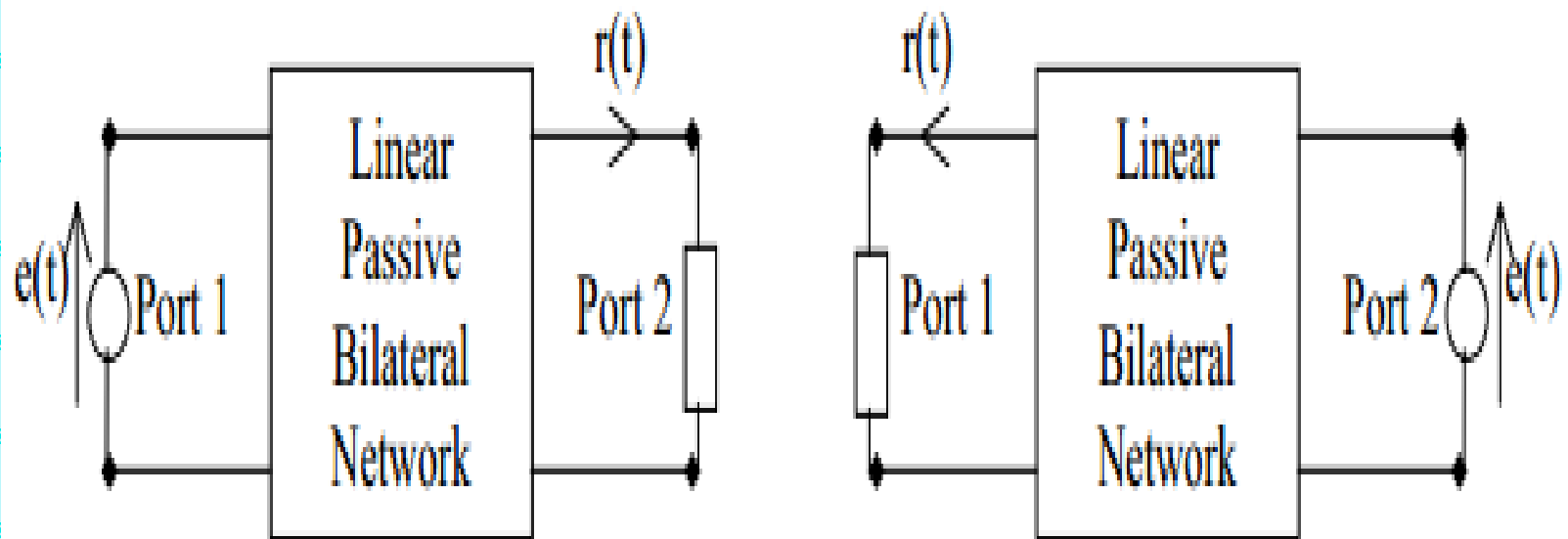
$$Z_{\text{eq}} = \frac{1}{\frac{1}{20} + \frac{1}{20}} = 10 \Omega \text{ (again same answer was obtained with Thevenin's Th}^{\text{m}})$$

$$\text{Hence current } I = \frac{85}{10 + 160} = 0.5 \text{ A}$$

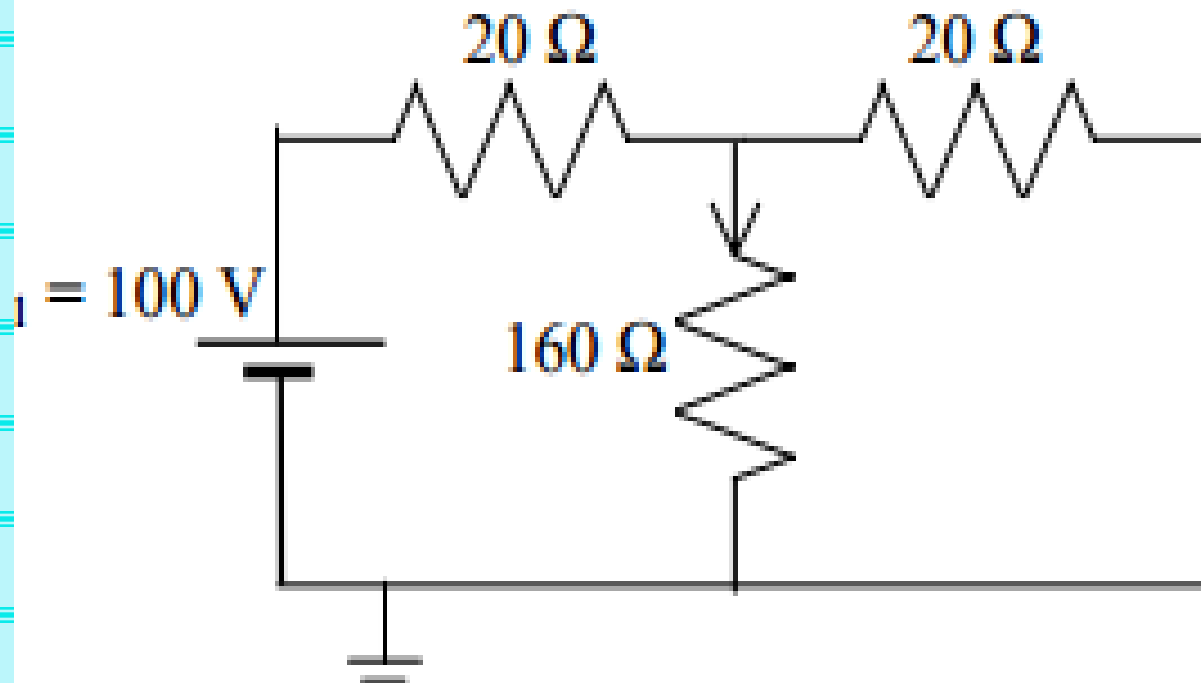
– Reciprocity Theorem

- ⌘ The reciprocity theorem is applicable only to single-source networks and states the following:
 - ⌘ The current I in any branch of a network, due to a single voltage source E anywhere in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.
 - ⌘ The location of the voltage source and the resulting current may be interchanged without a change in current

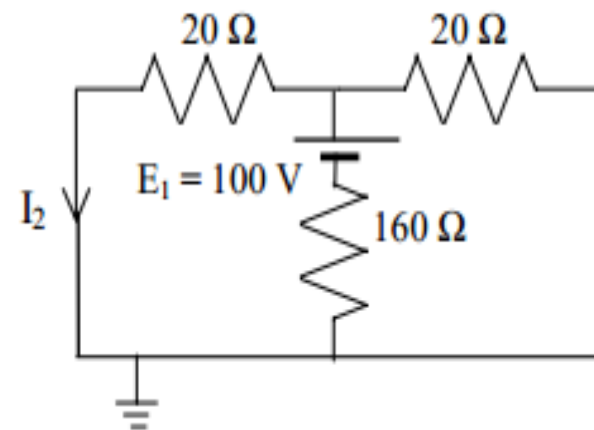
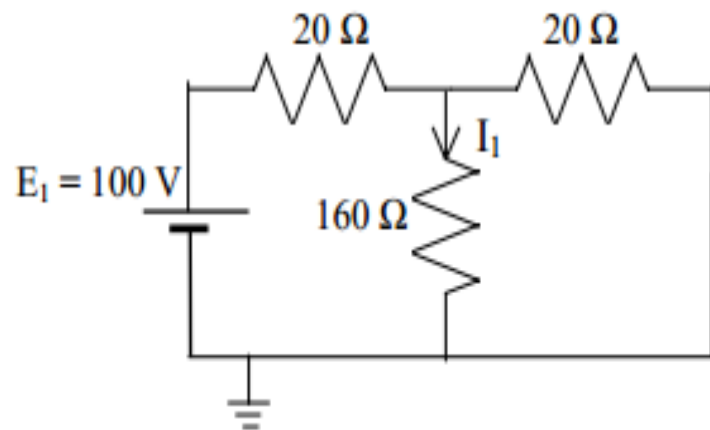
Reciprocity Theorem



EX -- Reciprocity Theorem



Solution



For the original circuit, current $I_1 = \frac{100}{20 + 160 // 20} \times \frac{20}{20 + 160} = \frac{2000}{37.778 \times 180} = 0.294 \text{ A}$

similarly for the new circuit, current $I_2 = \frac{100}{160 + 20 // 20} \times \frac{20}{20 + 20} = \frac{2000}{170 \times 40} = 0.294 \text{ A}$

It is seen that the identical current has appeared verifying the reciprocity theorem.